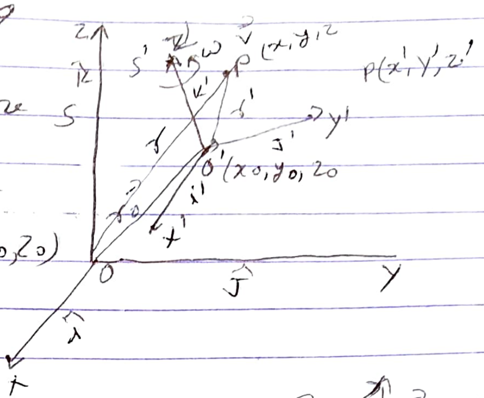
B.Sc-I

27/1/22

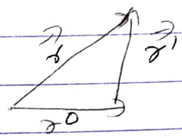
Fictitious force in non inertial frames having rotational motion →

Let P be a particle having co-ordinates (x, y, z) with respect to stationary frame S and (x', y', z') are its co-ordinates with respect to rotating frame S'. And (x_0, y_0, z_0) are co-ordinates O' w.r.t O



$$\vec{r}_0 = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



$$r' = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$

By using Δ law of vector addition

$$\vec{r} = \vec{r}_0 + \vec{r}'$$

$$x\hat{i} + y\hat{j} + z\hat{k} = x_0\hat{i} + y_0\hat{j} + z_0\hat{k} + x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$

Differentiating on both side w.r.t (t)

$$\dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} = \dot{x}_0\hat{i} + \dot{y}_0\hat{j} + \dot{z}_0\hat{k} + \dot{x}'\hat{i}' + \dot{y}'\hat{j}' + \dot{z}'\hat{k}') \quad \left(\dot{x} = \frac{dx}{dt}\right)$$

$$+ x'\frac{di'}{dt} + y'\frac{d\hat{j}'}{dt} + z'\frac{d\hat{k}}{dt})$$

$$\vec{V} = \vec{V}_0 + \vec{V}' + \left(x'\frac{di'}{dt} + y'\frac{dJ}{dt} + z'\frac{dk}{dt}\right) \quad \text{——} \ (1)$$

consider the relation

$$\vec{V} = \vec{\omega} \times \vec{R}$$

$$\frac{d\vec{R}}{dt} = \omega \times R$$

we can write

$$\frac{di'}{dt} = \bar{\omega} \times \hat{i}', \quad \frac{d\hat{j}'}{dt} = \bar{\omega} \times \hat{j}', \quad \frac{dk'}{dt} = \bar{\omega} \times \hat{k}'$$

Putting (1)

we get

$$\vec{v} = \vec{V_0} + \vec{V'} + \omega \times (x'\hat{i}' + y'\hat{j}' + z'\hat{k}')$$

$$\vec{v} = \vec{V_0} + \vec{V'} + \vec{\omega} \times r'$$

↳ Translational velocity S' frame

$\vec{v}$ is velocity of particle at P w.r.t S

$\vec{v}'$ is velocity of particle at P w.r.t S'

$\vec{\omega}$ = Angular velocity of rotating frame

If S' is not under linear motion then $\bar{V}_0 = 0$

$$\vec{v} = \vec{V'} + \vec{\omega} \times \vec{r}$$

$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{d\vec{r}}{dt}\right)_{rotating\ frame} + \vec{\omega} \times \vec{r}$$

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This relation is true for any general vector $\vec{v}, \vec{a}$

Putting $\vec{r}$ as $\vec{v}$

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \left(\frac{d\vec{v}}{dt}\right)_{rotating} + \vec{\omega} + \vec{v}$$

Put $\vec{v} = \vec{v}' + \vec{\omega} \times \vec{r}$ on RHS

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \frac{d}{dt}(\vec{v}' + \vec{\omega} \times \vec{r})_{rot.} + \vec{\omega} \times (\vec{v}' + \vec{\omega} \times \vec{r})$$

$$\left(\frac{dv}{dt}\right)_{fix} = \left(\frac{d\vec{v}'}{dt}\right)_{rotating} + (\dot{\vec{\omega}} \times \vec{r})_{rotate} + \vec{\omega} \times \vec{v}' + (\vec{\omega} \times \vec{v}')$$

$$\vec{a} = \vec{a}' + (\dot{\vec{\omega}} \times \vec{r}) + 2(\vec{\omega} \times \vec{v}) + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{a}' = \bar{a} - (\dot{\bar{\omega}} \times \vec{r}) - 2(\vec{\omega} \times \vec{v}) - \bar{\omega} \times (\vec{\omega} \times \vec{r})$$

Multiply both sides by m (mass of particle

$$m\vec{a}' = m\vec{a} - m(\dot{\vec{\omega}} \times \vec{r}) - 2m(\vec{\omega} \times \vec{v}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$m\vec{a}'$ = Force on particle as seen in S'

ma = Real force from S

$m(\dot{\bar{\omega}} \times \vec{r})$ = Transverse Force

$-2m(\vec{\omega} \times \vec{v})$ = coriolis force

$-m\vec{\omega} \times (\bar{\omega} \times \bar{r}) \rightarrow$ centrifugal force